# The Antiproton Component of the Primary Cosmic Ray Flux*

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## ABSTRACT

We have solved a Fokker-Planck diffusion equation for the diffusion and acceleration of cosmic rays. A time-independent solution is obtained by assuming that the contributions to the present intensity of particles injected at a uniform rate during all past times is equivalent to summing the intensities over all future time of particles injected at a given time. We have used continuous deceleration and fluctuations in acceleration to explain the energy spectrum. This gives us a general expression for a power law spectrum in which the exponent varies as

$$\gamma = 0.09 \ \ln \left(\frac{E}{m}\right) + 0.90,$$

thus allowing a very good fit to the experimental energy spectrum.

We then apply a previously developed expression for the production of antiprotons to obtain an injection spectrum. This and the above solutions are then applied to three extreme cases of possible origin. The resulting antiproton spectra depend upon whether the material encountered by primary cosmic ray protons is mostly inside or outside the regions of acceleration. The relative ratio of antiprotons and protons with E > 10 GeV is expected to be about $10^{-6}$ if the protons have passed through 2 - 3 gm/cm$^2$.

## I. INTRODUCTION

In the past, many efforts have been made to predict the intensity of the antiproton component of the primary cosmic ray flux.[1,2,3,4] The possibility of discovering the antiproton in the cosmic ray flux led to the first papers. Later attempts were usually made to explain the lack of observations of antiprotons in the primary cosmic ray flux. Most of the efforts were hindered by lack of any means of predicting the production cross section for antinucleons. Recently, this has changed to the extent that one can estimate with some confidence the order of magnitude of this cross section. The previous efforts considered only in a rough manner the problems of diffusion, acceleration, and origin of the antinucleons. In this paper, we hope to overcome the objections mentioned above.

We will first develop a theoretical framework for the origin of cosmic rays by solving the appropriate Fokker-Planck diffusion equation. We will then apply the results of this analysis to antiprotons under the assumptions of different regions of origin and types of acceleration. Finally, the results of this calculation suggest the possibility of using the antiproton cosmic ray flux to distinguish among several hypotheses for the origin and history of cosmic ray primaries.

## II. THE MODEL

We shall consider a slight modification of the usual supernova origin of cosmic rays.[2,5] To a large extent, this is a regrouping of many ideas from numerous sources into a general overall view of the processes involved. Several new and interesting results emerge that serve to give a more complete picture.

Consider a supernovae in which we have three regions. We shall use the word suprenovae in the generical sense of including all possible local sources that satisfy the conditions given. First, there will be a relatively dense central region (the core). Surrounding this in an approximately spherically symmetric region is matter ejected by the supernovae (the supernovae shell). This region will contain volumes of plasma that act as scattering centers. These turbulently moving centers will have a velocity distribution which will allow a statistical acceleration and deceleration to take place. The third region (the galaxy) is interstellar space within the galaxy, where the spectra of cosmic ray particles is assumed identical to that intercepted by the Earth. This model will only be applied to acceleration and diffusion of protons and to the subsequent production of antiprotons in proton-proton collisions. It shares the difficulty common to other models of being unable to explain the mechanism for acquiring the necessary energy for injection of heavy nuclei. However, it may be possible, within the framework of this model, to explain this in terms of cloud-cloud collisions. Although we will not consider heavy nuclei, we will make extensive use of information obtained from their isotopic abundances in the primary flux; namely, that cosmic rays have passed through approximately 2 - 3 $gm/cm^2$ of material.

The course of events in the lifetime of a cosmic ray proton is thus:

a. The particle is ejected from the core with an energy $E_{inj}$.
b. In the supernovae shell, the particle may be accelerated while diffusing, eventually diffusing out into interstellar space.
c. In interstellar space, the particle is further diffused and perhaps accelerated until it reaches the Earth.

For antiprotons, three limiting cases are considered:

Case I.

a. The antiprotons are created with an energy $E_{inj}$ in the supernovae shell by collisions of the cosmic ray protons having a spectrum similar to that observed at Earth, with supernovae gas nuclei (protons).
b. The antiprotons are accelerated in the process of diffusing out of the supernovae shell into the galaxy.
c. The antiproton flux is then diffused throughout the galaxy, but without further acceleration or production of additional antiprotons.

In this case, all the matter encountered by cosmic ray protons during their history of acceleration and diffusion is assumed to be in the supernovae shell. Acceleration of protons and antiprotons occurs only in the supernovae shell.

Case II.

a. The proton is injected into the galaxy from the supernovae shell to be accelerated while diffusing in the galaxy.

b. This proton flux interacts with interstellar matter to produce antiprotons.

c. The antiprotons are then accelerated and diffused in the galaxy.

In this case, all the matter encountered by cosmic ray protons is assumed located in interstellar space. Acceleration of protons and antiprotons occurs only in galactic space.

Case III.

a. The proton component is accelerated within the supernovae shell but without traversing a large amount of matter ($<<3$ gm/cm$^2$).

b. These protons, while diffusing throughout the galaxy without further acceleration, interact with interstellar matter to produce the antiprotons.

c. The antiprotons then diffuse through galactic space without acceleration.

The most likely case would be some combination of the above cases.

## III. EQUATION DESCRIBING PARTICLES IN A DIFFUSION REGION

The equation that describes the differential concentration spectrum of cosmic rays in a diffusion region can be written as[2]

$$\frac{\partial n}{\partial t} - D\nabla^2 n + \frac{\partial}{\partial E}\left( \frac{<\Delta E>}{\Delta t} n \right) - \frac{1}{2} \frac{\partial^2}{\partial E^2}\left( \frac{<\Delta E^2>}{\Delta t} n \right) + \frac{n}{T} = Q \qquad (1)$$

where the integral of n over the volume of the source is the density of particles.

The first two terms on the left-hand side of Eq. (1) are the usual diffusion terms. We have assumed that the diffusion coefficient, D, is not a function of E, $\vec{R}$, t. In doing this, we have ignored possible effects caused by the change of structure (and scales) of the inhomogenetics in the diffusion region. In an exploding shell, D may vary; however, the time over which we shall apply this equation is short compared to the lifetime of the shell. This allows us, as an approximation, to ignore the time dependence of D. Furthermore, we shall apply Eq. (1) in an energy range that is low enough so that we can assume D is not energy dependent.

In the third and fourth terms of Eq. (1) we account for the acceleration and energy loss in collisions which produce a continuous change in the cosmic ray particle energies. The third term arises from the mean statistical energy change of cosmic ray particles. The fourth term takes into account the statistical fluctuations in the energy change. These two terms are of the type found in Fokker-Planck equations. We will make the usual assumption that other terms which might be added to Eq. (1) may be neglected.[6]

The last term comes from the removal interaction processes in which the particles interact with the medium to produce particles other than the particle in question. We should remark here that this equation, as written, is valid only for one type of particle. We could easily extend it to include many particles by proper subscripting and summation, and changing the removal interaction term and the source term. Here T is the mean lifetime with respect to the removal interaction. Q is, obviously, the source term (see appendix B).

We shall consider the case where

$$Q = q(E, \vec{r})\ \delta(t - t_\circ) \tag{2}$$

We can solve Eq. (1) by applying the Mellin transformation

$$g(s, \vec{r}, t) = \int_0^\infty E^{s-1}\ n(E, \vec{r}, t)\ dE \tag{3}$$

and

$$x(s, \vec{r}) = \int_0^\infty E^{s-1}\ q(E, \vec{r})\ dE \tag{4}$$

to Eq. (1) to obtain

$$\frac{\partial g}{\partial t} - D\nabla^2 g - [(s-1)a + (s-1)(s-2)b] + \frac{g}{T} = x\delta(t - t_\circ) \tag{5}$$

where we have set

$$\frac{<\Delta E>}{\Delta t} = aE \ ,$$

$$\frac{<(\Delta E)^2>}{\Delta} = 2bE^2 \ .$$

We can solve Eq. (5) by standard methods [2,7] to find

$$n(E,\vec{r},t) = \frac{\nu q_o E_o^{-1} \left[ \left(\frac{E_m}{E_o}\right)^{(s_o-1-\nu)} - 1 \right] \left(\frac{E_o}{E}\right)^{s_o} \exp(A)}{(s_o-1-\nu)\ (4\pi b\tau)^{1/2}\ (4\pi D\tau)^{3/2}\ [1-(E_o/E_m)^{\nu}]} \tag{6}$$

where

$$A = (s_o-1)a\ \tau - \frac{|R|^2}{4D\tau} + (s_o-1)\ (s_o-2)b\ \tau - \frac{\tau}{T}\ ,$$

$$s_o = \frac{1}{2b\tau}\ \ell n\ (E/E_o) + \frac{3b-a}{2b}\ ,$$

$$\tau = t - t_o\ ,$$

$$R = \vec{r} - \vec{r}_o\ .$$

We have assumed that the source has a power law spectrum defined by

$$q(E,\ \vec{r})dE = \frac{\nu q_o\ \left(\frac{E_o}{E}\right)^{\nu} \frac{dE}{E}\ \ \delta(\vec{r} - \vec{r}_o)}{[1 - (E_o/E_m)^{\nu}]} \qquad \text{for } E_o \le E \le E_m$$

$$= 0 \text{ for } E < E_o \text{ and } E > E_m. \tag{7}$$

In the above $E_o$ is the minimum injection energy and $E_m$ is the maximum injection energy.

Since the intensity of cosmic radiation in the galaxy seems to be time independent, we are interested in a steady state solution. We wish to sum the contributions to the present cosmic ray concentration of particles injected at all past times. If the rate of supernovae explosions and generation of plasma turbulence has been uniform in past time, this is equivalent to summing over the history of the acceleration and diffusion process for a group of particles which had been injected at the same time. This equivalence assumption is very important since it allows us to directly utilize the comparatively simple mathematical analysis outlined above to obtain directly measurable relations. To do this we evaluate the integral

$$\eta(E, \vec{r}) = \int_0^\infty n(E, \vec{r}, \tau)\, d\tau \, . \tag{8}$$

By the use of Laplace integration,[8]

$$\int_0^\infty \Phi(x)\, e^{kh(x)}\, dx \sim \Phi(a)\, e^{kh(a)} \sqrt{\frac{-\pi}{2kh''(a)}} \quad ,$$

where a is the root of $(dh/dx) = 0$ and $h'' = d^2h/dx^2$ .

We can evaluate Eq. (8) to find

$$\eta(E,\vec{r}) = \frac{\nu\, q_o}{K\, E_o} \left[ \left(\frac{E_m}{E_o}\right)^{(\gamma_1-\nu-1)} - 1 \right] e^{-\frac{(a-b)}{b} - \frac{|R|^2}{4D\tau_o} - \frac{\tau o}{T}} \left(\frac{E_o}{E}\right)^{\gamma} \tag{9}$$

where

$$K = 2^4 \pi^{3/2} (\gamma_1-\nu-1) b^{1/2} D^{3/2} \left[1-\left(\frac{E_o}{E_m}\right)^{\nu}\right] \tau_o$$

$$\gamma_1 \equiv \frac{1}{2b\tau_o} \ \ln\left(\frac{E}{E_o}\right) + \frac{3b-a}{2b} \quad ,$$

$$\gamma \equiv \frac{a(a-b)}{16b^2} \ \ln\left(\frac{E}{E_o}\right) + \frac{3b-a}{2b} \quad ,$$

$$\tau_o \equiv \frac{4b}{a(a-b)} \quad .$$

We find that in this integration, the error in dropping higher order terms is approximately 2%. The mean number of cosmic ray particles per unit volume in the source, a sphere of radius $R_o$, is given by

$$n(E) = \int \eta(E,r)\, dV = \int_0^{R_o} \eta(E,r)\; 4\pi r_o^2 dr_o$$

or letting $x = \dfrac{r_o^2}{4D\tau_o}$ ,

$$n(E)dE = \frac{\nu q_o \tau_o^{1/2} \Phi(r) \exp[-\frac{\tau_o}{T} - \frac{(a-b)}{a}]\ [(\frac{E_m}{E_o})^{(\gamma_1-\nu-1)} -1]}{\pi^{1/2}\ b^{1/2} (\gamma_1-\nu-1) \qquad [1-(\frac{E_o}{E_m})^{\nu}]} (\frac{E_o}{E})^{\gamma-1} \frac{dE}{E} \tag{10}$$

where

$$\Phi(r) = \int_0^{\frac{R_o^2}{4D\tau_o}} x^{1/2} \exp\left(\frac{-|r-r_o|^2}{4D\tau_o}\right) dx.$$

In integrating Eq. (10) from E to ∞ to obtain the integral concentration, we note that the result is dominated by the behavior of the integrand near the lower limit of integration, E. Therefore, we may approximate the result by holding $\gamma$ fixed in the integrand and equal to its correct value at E. Integrating, we obtain:

$$N(E) = \int_E^{\infty} n(E)\, dE$$

$$= \frac{\nu q_o \tau_o^{1/2} \Phi(r) \exp[-\frac{\tau_o}{T} - \frac{(a-b)}{a}]}{\pi^{1/2} b^{1/2}\ (\gamma_3-\nu)(\gamma-1)[1-(E_o/E_m)^{\nu}]} [(\frac{E_m}{E_o})^{\gamma_3-\nu} -1] (\frac{E_o}{E})^{\gamma-1} \tag{11}$$

where

$$\gamma_3 = \frac{a(a-b)}{8b^2}\ \ell n\ (\frac{E}{E_o}) + \frac{b-a}{2b} \quad .$$

If we had used a monoenergetic source term [$q = q_o \ \delta(E - E_o)$], we would have found

$$N(E) = \frac{q_o \tau_o^{1/2} \quad \Phi(r)}{\pi^{1/2} b^{1/2} (\gamma - 1)} \ \exp\left[-\frac{\tau_o}{T} - \frac{(a-b)}{b}\right] \left(\frac{E_o}{E}\right)^{\gamma-1} \tag{12}$$

To obtain the integral energy flux, J(E), we note that

$$J(E) = \frac{cN(E)}{4\pi} \left(\frac{\text{particles}}{\text{cm}^2 \ \text{sec} \cdot \text{sr.}}\right) \tag{13}$$

## IV. THE FOKKER - PLANCK COEFFICIENTS

Fermi has shown that the change in energy of a particle upon being "scattered" by a moving center is given by[9]

$$\Delta E = E[\Gamma^2 \ (1 + 2\beta \ B \cos \theta + B) -1] \tag{14}$$

where $\beta c = v$ = velocity of the particle, $Bc = V$ = velocity of the scattering center and $\theta$ is the angle between v and V.

The collision probability, $\Psi \ (\vec{v}, \cos \theta) \ d\vec{v} \ d \ (\cos \theta)$, is proportional to the relative velocity given by [10]

$$\frac{v_r}{c} = \frac{(\beta^2 + B^2 - 2\beta \ B \cos \theta - \beta^2 \ B^2 \sin^2 \theta)^{1/2}}{1 - \beta \ B \cos \theta} \tag{15}$$

and to the velocity distribution, $f(\vec{V})$, of the scattering centers:

$$\Psi \ d\vec{V} \ d \ (\cos \theta) = \frac{v_r \ f(\vec{V}) \ d\vec{V} \ d \ (\cos \theta)}{\int\int v_r \ f \ (V) \ dV \ d \ (\cos\theta)} \tag{16}$$

We will assume that $\cos \theta$ is isotropically (uniformly) distributed.

Laster, Lenchek and Singer[11] have shown that there is an important deceleration term when one considers magnetically turbulent scattering centers which are receding from each other with a velocity $\lambda V_e/R$ in the case of spherical expansion from a common center. Here $V_e$ is the expansion velocity, $\lambda$ is the mean free path between centers and R is the radius of the expansion. (This mechanism was first proposed by Ginzburg, Pikel'ner and Shklovskii[12] in one of the original articles dealing with supernovae origin of cosmic rays.) Following the Russian group, we will write the deceleration term as

$$- \frac{v\, V_e\, \lambda}{c^2 R}\, E \tag{17}$$

Laster, Lenchek and Singer have shown that for a continually weakening magnetic field, B, that there is a betatron deceleration. In this case the deceleration term would be $(2v\, V_e\, \lambda)/(3c^2 R)E$, where $V_e$ is now the rate at which the Larmor radius is increasing.

Recall that we have defined

$$a = \frac{\langle \Delta E \rangle}{E \Delta t} \quad , \quad 2b = \frac{\langle (\Delta E)^2 \rangle}{E^2 \Delta t} \quad .$$

If we combine Eqs. (14) and (17) and expand the result and Eq. (15), keeping terms up to order of $B^3$, we find that

$$a \approx \frac{8}{3} \bar{B}^2 - 2k + \frac{2}{3} k\, \bar{B}^2 \ , \tag{18}$$

$$2b \approx 2(k^2 - 4k\, \bar{B}^2) + \frac{8}{3} \bar{B}^2 \quad , \tag{19}$$

where

$$k \equiv \frac{v\, V_e \lambda}{c^2\, R} \quad \text{or} \quad \frac{2v\, V_e\, \lambda}{3\, c^2\, R}$$

$$\langle B^2 \rangle \equiv \int B^2\, f(\vec{V})\, d\vec{V} \quad .$$

Here we have used

$$\langle (\Delta E)^n \rangle = \int \int (\Delta E)^n\, \Psi\, dV\, d(\cos\, \theta).$$

## V. SIZE ESTIMATES

In order to calculate the fluxes in which we are interested, we must have some estimate of the extent of the supernovae shell. We will find this by calculating that point at which the density of cosmic rays coming from the shell is equal to the density of cosmic rays in galactic space (thus assumed to be equal to the intensity at the Earth). At the perimeter of the supernovae shell the diffusion characteristics will be the same as those of interstellar space. It is in this region that we can say that the supernovae shell ends. With this in mind, recall that for a source at the origin with unit density at the time t = 0, we can write

$$N_{C.R.}{}^{\circ}(R,t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left(\frac{-R^2}{4Dt}\right)$$

$$= \left(\frac{3}{4\pi Lct}\right)^{3/2} \exp\left(\frac{-3R^2}{4Lct}\right) \tag{20}$$

where

$$D = \frac{Lc}{3} \quad ,$$

L = transport mean free path in interstellar space.

If now we assume a generation rate for cosmic rays, $G_{C.R.}$, with the properties

$$G_{C.R.} = 0 \text{ for } t < 0$$

$$= S_{C.R.} = \text{constant for } t > 0$$

we have for the density of cosmic rays (at a distance R from the source) after a time t,

$$N_{C.R.}(R) = \int_o^t G_{C.R.} N_o(R,t')\, dt' \tag{21}$$

Then

$$N_{C.R.}(R) = S_{C.R.} \left(\frac{3}{4\pi Lc}\right)^{3/2} \int_o^t t'^{-3/2} e^{-a/t'} dt' \tag{22}$$

where we have taken $a = 3R^2/4Lc$. Let $x = a/t$, so that

$$N_{C.R.}(R) = \frac{S_{C.R.}}{\sqrt{a}} \left(\frac{3}{4\pi Lc}\right)^{3/2} \int_{a/t}^{\infty} x^{-1/2} e^{-x} dx$$

$$= \frac{3}{4\pi^{3/2}} \frac{S_{C.R.}}{LcR} [\Gamma(1/2) - \gamma(1/2, a/t)] \tag{23}$$

and $\gamma$ is the incomplete gamma function. The asymptotic limit of $\Gamma(\frac{1}{2}) - \gamma(\frac{1}{2}, \frac{a}{t})$ as $t \to \infty$ is approximately 1.77. We shall use this in our calculations. It is known that $N_{C.R.} \simeq 10^{-10}/cm^3$. If we then take the limit of the supernovae shell as that point at which the density of cosmic rays is equal to that of interstellar space, we obtain the radius of a typical source. This is what we shall do by setting Eq. (23) equal to $10^{-10}/cm^3$. We will take $L \simeq 10^{20}$ cm[1] and $S_{C.R.} \simeq 2 \times 10^{40}$/sec.[3] Thus $R \simeq 5pc = 1.5 \times 10^{10}$ cm.

The average amount of matter traversed, x (in $gm/cm^2$) is given by

$$x = \bar{\rho}\ c\ \tau_{eff}$$

where $\bar{\rho}$ is the mean density. The effective mean lifetime, $\tau_{eff}$, is given in Appendix A to be

$$\tau_{eff} \approx \tau_{leak} \approx \frac{3h^2}{2\lambda c} \ ,$$

$$\lambda \approx \frac{1}{\pi S_c^2 \ n_c} (1 + R_L^2/S_c^2),$$

or

$$x = \frac{3\bar{\rho}\ \pi\ S_c^2\ n_c}{2[1 + (R_L^2/S_c^2)]}\ h^2 \quad , \tag{24}$$

where

$S_c$ = mean diameter of scattering centers,

$n_c$ = mean density of scattering centers,

$R_L$ = Larmor radius of particles within the scattering centers,

2h = smallest dimension of diffusion region.

Let us now consider each case and how the above x and ρ are related to it.

Case I.

(All the acceleration and most of the material traversed is within the supernovae shell.)

$x_{SN}$, the amount of material traversed in the supernovae shell, is some function of the total amount of material traversed, we take this to be 2.5 $gm/cm^2$. We see that the amount traversed in the galaxy, $x_G$, is just $X_G = 2.5\ (gm/cm^2) - x_{SN}$. From Eq. (24) we can find approximately the effective dimension of the galaxy. When we use the values for $S_c$, $n_c$, and $R_L$ given in Appendix A and take $\bar{\rho} \simeq 10^{-2}$ $atom/cm^3$ we obtain the relation between $X_G$ and h shown in Fig. 1.

Case II.

(All the acceleration and all of the material traversed is in the galaxy.)

Here the entire 2.5 $gm/cm^2$ of material traversed is located in galactic space. Again we will use the values given in Appendix A and take $\bar{\rho} \approx 10^{-2}$ $atom/cm^3$ to find that $h \simeq 12.5$ kpc.

Case III.

(All the acceleration is within the supernovae shell, but most of the material traversed is in the galaxy.)

Referring again to Fig. 1, $x_G \simeq 2.5$ $gm/cm^2$, so $h \simeq 12.5$ kpc.

## VI. THE PROTON SPECTRUM

### A. Proton Spectrum When All the Acceleration is Within the Supernovae Shell.

The equations that we have developed above apply to the diffusion process within the acceleration region. However, after escaping from this region, the particles must diffuse through the galaxy. Although, in this extreme case, they are not further accelerated, they will have collisions with interstellar scattering centers. We can describe this steady-state process by the equation[13]

$$\frac{\lambda}{3} \nabla^2 \Phi - \frac{1}{\lambda} \frac{\partial \Phi}{\partial n} = -s \, \delta(n) \tag{25}$$

where

$\Phi$ = scalar flux,

$\lambda$ = mean free path,

s = source density (number of particles injected per sec., per unit volume),

n = number of collisions with scattering centers.

For the case of spherical symmetry, we find that in the fundamental mode with a point source at the center

$$\Phi(\vec{r},n) = \frac{\lambda s}{2a^2 r} \sin\left(\frac{\pi r}{a}\right) e^{-n/N_1} \tag{26}$$

where

$$N_1 = \frac{3a^2}{\lambda^2 \pi^2},$$

$a$ = radius of spherical region (galaxy),

$s$ = strength of point source.

The total number of particles in the source volume per unit time is

$$A(n, a) = \int_0^a \Phi(\vec{r}, N)\ 4\pi r^2\ dr = 2\ \lambda\ s\ e^{-n/N_1} \tag{27}$$

Then the integral flux is given by

$$J(E) = \frac{1}{4\pi} \int_n^\infty A(n', a)\ dn' = \frac{3a^2}{2\pi^3}\ \frac{s}{\lambda}\ e^{-n/N_1} \tag{28}$$

In the above analysis we have assumed that all of the particles came from a single "prototype" supernovae. Thus we can use Eq. (11) for the number per unit volume in the supernovae to obtain for the source term

$$s = N(E)\ V_{sN}\ \frac{Q}{V_{Gal}} \tag{29}$$

where Q is the rate of occurrence of supernovae. We will use Zwicky's[14] result of one supernovae per 360 years.

We can estimate the number of collisions with the gas clouds, n, by noting that the collision rate is given by $c/\lambda$ and the mean diffusion time is given by $\tau \approx \frac{3a^2}{2\lambda c}$;

$$n = \frac{c}{\lambda} = \frac{3a^2}{2\lambda^2}$$

and

$$\frac{n}{N_1} = \frac{\pi^2}{2} \quad .$$

The minimum injection kinetic energy of the proton is a few hundred MeV, and $E_o$, the minimum total energy of the proton, is to a good approximation, equal to $m_p$. But this is also very close to the maximum injection energy. We are then led to consider the case of a monoenergetic source term.

If we write $V_{sN} = 4/3\ \pi\ R_s^3$ and $V_{Gal} = 4/3\ \pi\ R_G^3$, we find, combining Eqs. (12), (28), and (29), that the proton spectrum at the Earth would be

$$J(E) = \frac{3Q\ R_s^3\ q_o \tau_o^{1/2} \Phi(r)}{2\pi^{7/2}\ b^{1/2}\ R_G\ \lambda(\gamma-1)} \quad \exp\left[-\frac{\tau_o}{T} - \frac{(a-b)}{a} - \frac{\pi^2}{2}\right] \ (E_o/E)^{\gamma-1} \tag{30}$$

We can only obtain a satisfactory fit to the experimental data for the energy spectrum when we assume a continuous deceleration and have $a \approx -.9\ b$. Here we have assumed that the deceleration is mainly due to the expansion of the supernovae shell. Note that in doing this, we have a balancing of continuous acceleration against deceleration. While deceleration dominates, an acceleration process must also be present. Ginzburg and Syrovatskii[5] (P. 324) did not consider the possibility of negative "a" when they reached their conclusion that fluctuations would not play an important role in overcoming the difficulty that none of the solutions of the Fokker-Planck equation appear to agree with the observed spectrum. Our assumption of negative "a" not only resolves this difficulty, allowing a good fit to the observed spectrum, but follows for a reasonable choice of astrophysical parameters, as indicated above. In the case of deceleration dominating ("a" negative), the primary cosmic ray spectrum is entirely attributable to the consequences of statistical fluctuations.

When we apply the above results to the measured energy spectrum, we obtain the results shown in Figs. 2 and 3. The experimentally suggested curve of Wolfendale[15] for the variation of the exponent with energy is in good agreement with our result of $\gamma = 0.09 \ \ell n(E/m_p) + 0.9$. The shaded areas in Fig. 3 are those proposed by H. Bradt, et al.,[16] and S. I. Nikol'skii.[17] We have used $q_o$ as a free parameter to adjust the normalization. To obtain an estimate of $\langle B^2 \rangle$ we have used $\langle B^2 \rangle \approx 6 \times 10^{-10}$, the value appropriate to the Orion Nebulae.[18]

B. The Proton Spectrum When All the Acceleration Is Within the Galaxy.

Here we shall assume that the scattering centers are the interstellar gas clouds with $\langle B^2 \rangle \approx 2 \times 10^{-10}$. Again we shall take $a \approx -.9b$, i.e. deceleration dominates. But, for the galactic case, the most likely cause for negative "a" is betatron deceleration. Thus, we have the same results as those of Section VI-A.

## VII. THE ANTIPROTON SPECTRUM

We shall now calculate the antiproton spectrum for the three cases described in Section II.

If we now use the definition and expression for the differential injection spectrum given in Appendix B, we obtain the results shown in Fig. 4 with an assumed density of $\rho = 10^{-2}$ gm/cm$^3$. We have used the experimentally measured spectrum since, according to Eqs. (11) and (30), the spectrum in the supernovae shell and near the Earth should be similar in shape. The differential antiproton spectrum was found by direct numerical integration of the following equation

$$\left(\frac{dq}{dp}\right) dp = \rho \, dp \int_{E_{th}}^{\infty} C \frac{d\sigma(E)}{dp} \, M(E) \, dE. \tag{31}$$

We want to fit the results with an expression of the form

$$q(E)dE = \frac{\nu q_o \left(\frac{E_o}{E}\right)^{\nu} \frac{dE}{E}}{\left[1-\left(\frac{E_o}{E_m}\right)^{\nu}\right]} \quad \text{for } E_o \le E \le E_m$$

$$= 0 \text{ for } E < E_o \text{ and } E > E_m,$$

where $q_o$, $E_o$, $E_m$, and $\nu$ are adjustable parameters. We obtain a good fit and preserve equal areas under the curves when $q_o \approx 4.6 \times 10^{-32}$ ($\bar{p}$/cm$^3$ sec), $E_o = 1$ GeV, $E_m = 4 \times 10^3$ GeV and $\nu = 1.56$.

Case I.

(All the acceleration and most of the material traversed is within the supernovae shell.)

If the total mass within the supernovae shell is 10 $m_{\odot}$, the density is about 10 hydrogen atoms/$cm^3$. We chose the mean free path, $\lambda$, to be approximately $0.7 \times 10^{15}$ cm so that the average amount of material traversed by a proton is 2.5 gm/$cm^2$. These choices are consistent with the astrophysical parameters appropriate to a supernovae shell (See Sec. IV and Appendix A). We now use the above results for the injection spectrum, but with the normalization and results given for the Case I differential proton spectrum in Eq. (31), to obtain the antiproton spectrum shown in Fig. 5. The power-law slope is different than that for the proton spectrum because $\nu$ is small and $\frac{E_m}{E_o} \approx 4\times 10^3$ for antiproton injection. This results in an appreciable energy dependence of the coefficient, $\frac{[(\frac{E_m}{E_o})^{\gamma_3-\nu}-1]}{\gamma_3-\nu}$, in Eq. (11) for the antiproton spectrum.

Case II.

(All the acceleration and most of the material traversed is in galactic space.)

With a change in $q_o$ we can use the results obtained in Case I (Sec. VII and Fig. 5) for the injection spectrum. The density of matter is approximately $10^{-2}$ hydrogen atom per $cm^3$. Thus $q_o$ will be decreased by a factor $10^{-3}$. This is consistent with passing through 2.5 gm/$cm^2$ before leaking out of the galaxy. If we now insert this information and the values for the astrophysical quantities into Eq. (30), but with the power-law solution [Eq. (11)], we obtain the antiproton flux shown in Fig. 6.

### Case III.

(The production and diffusion of antiprotons within the galaxy without acceleration.)

The injection spectra will be the same as in Case II, and is shown in Fig. 4. Again we can use Eq. (28) and the values of the astrophysical parameters to obtain the antiproton spectrum shown in Fig. 7. Since the antiprotons are produced and diffused in a region where no acceleration takes place, the shape of the injection spectrum is preserved. It should be noted that this antiproton spectrum decreases much more rapidly with increasing energy than for Cases I and II.

## VIII. DISCUSSION AND CONCLUSIONS

We have made the assumption that the cosmic ray concentration (and thus flux) can be described by a Fokker-Planck diffusion equation [Eq. (1)] and that an average over the history of a typical group of injected particles gives the time independent solution of direct physical interest. It is then shown that the observed spectrum is the result of continuous deceleration and fluctuations in acceleration. What we are observing, then, is the result of the fluctuations. We have applied this model to various possible regions of origins of the proton and antiproton components. It is found that the model gives a remarkably good fit to the observed integral energy spectrum of primary protons (Fig. 2), with a power-law dependence in which the exponent itself is a weak function of the energy:

$$\gamma = 0.09 \ \ell n \left(\frac{E}{m}\right) + 0.9$$

The fit is not sensitive to the particular type of velocity distribution of scattering centers giving rise to the Fermi acceleration term and assumes an isotropic distribution of angles between the particle and scattering cloud velocity vectors. Also, the fit is relatively independent of whether the particles are accelerated in local regions, such as supernovae, or in the galaxy. The fit is not good at high energies (above approximately $10^7$ GeV). If, however, there is another component that contributes significantly only at energies above $10^7$ GeV and of slope

1.6, we could obtain the observed spectra. This component could be from cosmic rays that have enough energy to escape from their galaxies and then be accelerated on a megagalactic scale to the very high energy at which they are observed.[19]

We see that in comparing the case where all the acceleration is within the supernovae (Case I) to that in which all the acceleration is within the galaxy (Case II) that the shape of the final spectrum is the same if the Fokker-Planck coefficients are the same. The fact that the antiproton intensity is the same in Case I and Case II is due to the assumption in each case that the primary protons have passed through an average of 2.5 $gm/cm^2$ of hydrogen. We have assumed in the analysis that the mean free path, $\lambda$, is a constant. Above an energy of $10^6 - 10^7$ GeV this is no longer true. Thus we have not extended our results above this limit.

When one considers the case of no acceleration (Case III), simply antiproton creation and diffusion, one notices a marked change in the slope of the spectrum. Obviously, the high energy portion of the spectrum is enhanced by acceleration in Cases I and II. Previously, investigators have only considered creation and sometimes, in a very approximate manner, diffusion, which most closely corresponds to our Case III. While the antiproton spectrum shape and total intensity mainly depend upon $d\sigma/dp$ in p - p production collisions for Case III, they depend mostly upon acceleration processes for Cases I and II. Since the expected spectra contrast so sharply, it appears that when the antiproton spectrum is measured, a clear choice can be made between Case I or II vs. Case III.

We should state that although we have considered here supernovae shells, the results would also apply to other sources. Perhaps one should consider novae, moving envelopes of stars, etc.

In conclusion, we would like to emphasize that the final expression, Eq. (11), obtained in Section III for the flux is a general result that can be applied to any type of acceleration mechanism or velocity distribution of scattering centers where the processes have been going on at a constant rate for at least as long as the storage time for cosmic ray particles. The approximations made are very broad and are usually fulfilled when one is considering high energy cosmic ray phenomena.

## APPENDIX A

### The Lifetime for Leakage from a Region of Space

We will follow a method indicated by Morrison[20] to find the mean free path for a particle to leak out of a given region of space. One usually assumes that a particle will diffuse through space colliding with "magnetic clouds." Let $N_{leak}$ be the number of magnetic scatterings before a particle leaks out of a region. Then

$$\tau_{leak} = \text{lifetime for leakage} = \frac{N_{leak}\ \lambda_t}{c} \tag{A1}$$

where $\lambda_t$ is the mean free path between collisions for particles with a velocity c. Recall that in the case of a three-dimensional random-walk problem

$$\begin{aligned} \langle h^2 \rangle &= \text{mean sq. distance in h direction reached after N collisions} \\ &= \frac{2N\lambda^2}{3}\ . \end{aligned} \tag{A2}$$

Then we will have leakage when h $\approx$ 1/2 the smallest dimension of the region of space in question. Then

$$N_{leak} \approx \frac{3h^2}{2\lambda^2}$$

or

$$\tau_{leak} \approx \frac{3h^2}{2\lambda c} \tag{A3}$$

Beacuse we are using a simplified version of transport theory, we will use the transport mean free path

$$\lambda_t = \frac{\lambda}{1 - \langle\cos\theta\rangle} \tag{A4}$$

where λ = mean distance between deflections in the magnetic field, $\langle\cos\theta\rangle$ = mean cosine of angle of deflection. Recall that the Larmor radius $R_L$ is given by

$$R_L = \frac{m\, v_1}{Ze\, |B|}\ , \tag{A5}$$

or for protons in space[20]

$$R_L = \frac{3.52 \times 10^{-6}\ E_{GeV}}{|Z|\ B_{microgauss}}\ \text{(in light years)}.$$

$$= \frac{1.079 \times 10^{-6}\ E_{GeV}}{|Z| B_{\mu gauss}}\ \text{(in parsecs)}. \tag{A6}$$

If $R_L$ of the diffusing particle is small compared to the diameter of the magnetic cloud, $S_c$, then θ > π/2 and we have

$$\lambda_t \simeq \lambda \simeq \frac{1}{\pi S_c^{\ 2} n_c} \qquad (R_L \ll S) \tag{A7}$$

where

$$n_c = \text{density of diffusing clouds in space.}$$

If $R_L < S_c$, we have $\langle\theta\rangle \approx \frac{S}{R_L}$ and

$$\lambda_\ell \approx \frac{1}{\pi S_c^2 n_c} \left(\frac{R_L}{S_c}\right)^2 \tag{A8}$$

Then, following E. Parker,[21] we can write as a fair approximation

$$\lambda_t \approx \frac{1}{\pi S_c^2 n_c} \left(1 + \frac{R_L^2}{S_c^2}\right) \tag{A9}$$

We will take for the physical parameters $S_c = 30$ pc, $\bar{\rho} = 1.66 \times 10^{-24}$ gm/cm$^3$, $n_c = 3 \times 10^{-5}$ pc$^{-3}$,[22] and $B = 5 \times 10^{-6}$ gauss.[3]

## APPENDIX B

### The Production Rate

The differential rate at which antiprotons are produced in a given volume with momenta between p and p + dp is given by

$$\left(\frac{dq}{dp}\right) dp = dp \cdot \rho \int_{E_{th}}^{\infty} \frac{d\sigma(E)}{dp} \quad c\, n(E)\, dE \tag{B1}$$

where

$\rho$ = number of hydrogen atoms per unit volume in the region in question,

$\frac{d\sigma(E)}{dp}$ = differential cross-section for the production in a p - p collision of an antiproton of momentum p by a proton with laboratory energy, E,

C n(E) = differential energy spectrum of the primary cosmic ray spectrum,

$E_{th}$ = threshold energy for production of antiprotons.

The production rate is given by

$$q = \int_{o}^{p_{max}} \frac{dq}{dp}\, dp \tag{B2}$$

To obtain an expression for the differential cross-section $d\sigma/dp$, we will use the results of J. R. Wayland and T. Bowen:[23]

$$\frac{d^2\sigma}{dpd\Omega} = k\, T\, \mu_2'\, K_1 \left(\frac{\mu_2'}{T_o}\right) \exp\left(-\frac{\mu_1'}{T}\right) \left(1 + \frac{\mu_1'}{T}\right) \frac{E'p^2}{E}\ , \tag{B3}$$

where the primed quantities are in the center-of-mass system and

$T$ = "Longitudinal temperature,"

$T_o$ = "transverse temperature,"

$\mu_2^2 = (p_\perp^2 + m^2)$,

$\mu_1^2 = (p_\parallel^2 + m^2)$,

$p$ = momentum of produced secondary,

$m$ = mass of produced secondary,

$$k = \frac{2V_o}{h^3 m^2 c^4 T_o K_2 \left(\frac{mc^2}{T_o}\right)} ,$$

$V_o$ = interaction volume.

As we are working with high energies, $d^2\sigma/dpd\Omega$ decreases very rapidly with increasing $\theta$. Therefore, we can write $\cos\theta \simeq 1$ and $\sin\theta \simeq \theta$, and extend the range of integration over $\theta$ to $\infty$. Thus we have

$$\frac{d\sigma}{dp} = \int_o^\infty \frac{k\ T\ E'p^2}{E\gamma(1-\beta\frac{E}{p})} (p'^2\theta^2+\delta^2)^{1/2}\, K_1\left[\frac{(p'^2\theta^2+\delta^2)^{1/2}}{T_o\gamma(1-\beta\frac{E}{p})}\right] \exp\left(\frac{-\mu_1'}{T}\right)\left(1+\frac{\mu_1'}{T}\right)2\pi\theta d\theta. \tag{B4}$$

$$\frac{d\sigma}{dp} = \Phi\,\frac{T\gamma(E-\beta p)}{E}\, e^{-\mu_1'/T}\left(1+\frac{\mu_1'}{T}\right) \tag{B5}$$

where

$$\Phi = 2\pi\ T_o\ m^2 K_2\left(\frac{m}{T_o}\right) k ,$$

$$\mu_1'^2 = \gamma^2 (p-\beta E)^2 + m^2.$$

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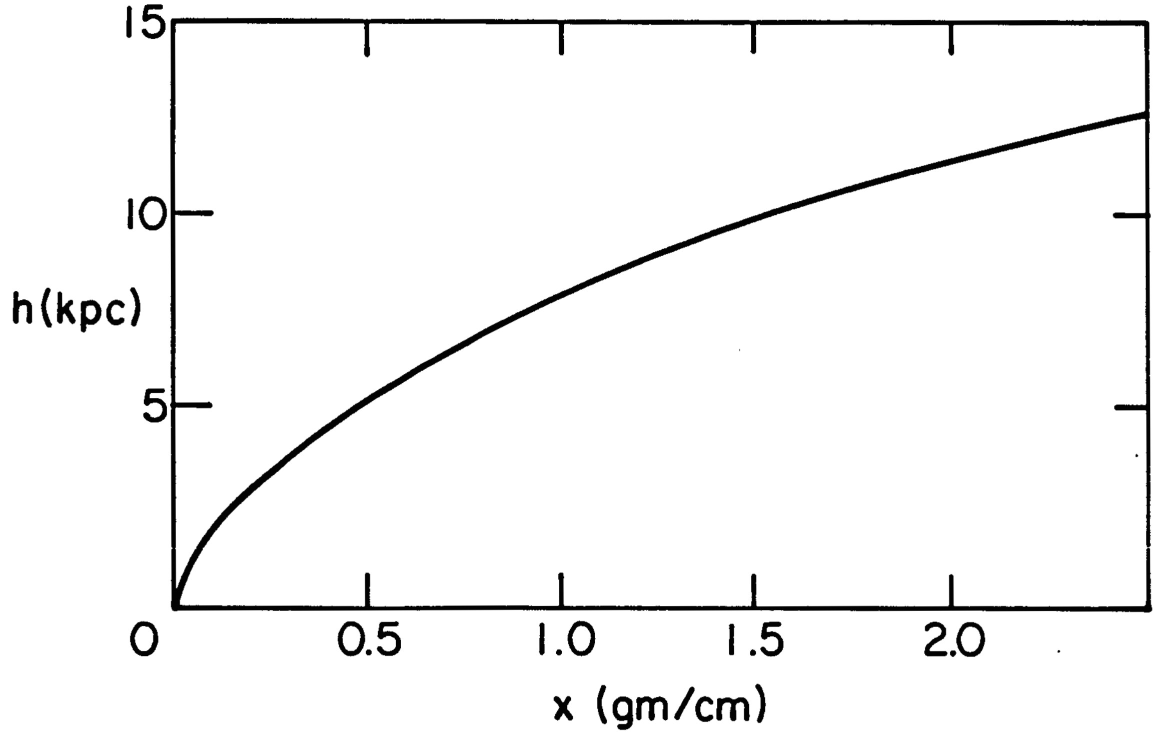


Fig. 1 THE DIMENSION OF THE GALAXY AS A FUNCTION OF THE AMOUNT OF MATERIAL TRAVERSED WHEN $\bar{\rho} = 10^{-2}$ atoms/cm$^3$.

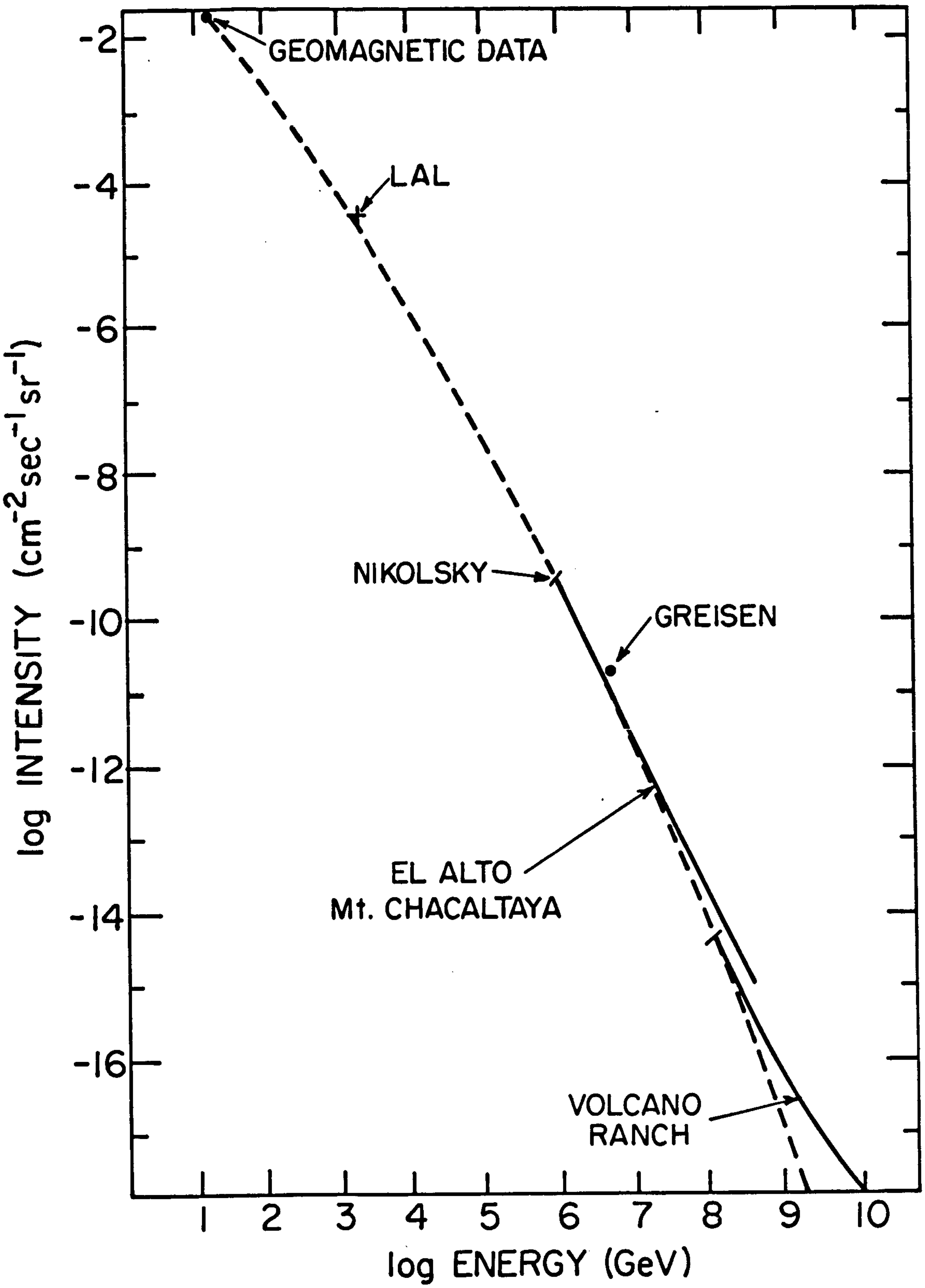


Fig. 2 THE PRIMARY ENERGY SPECTRUM.

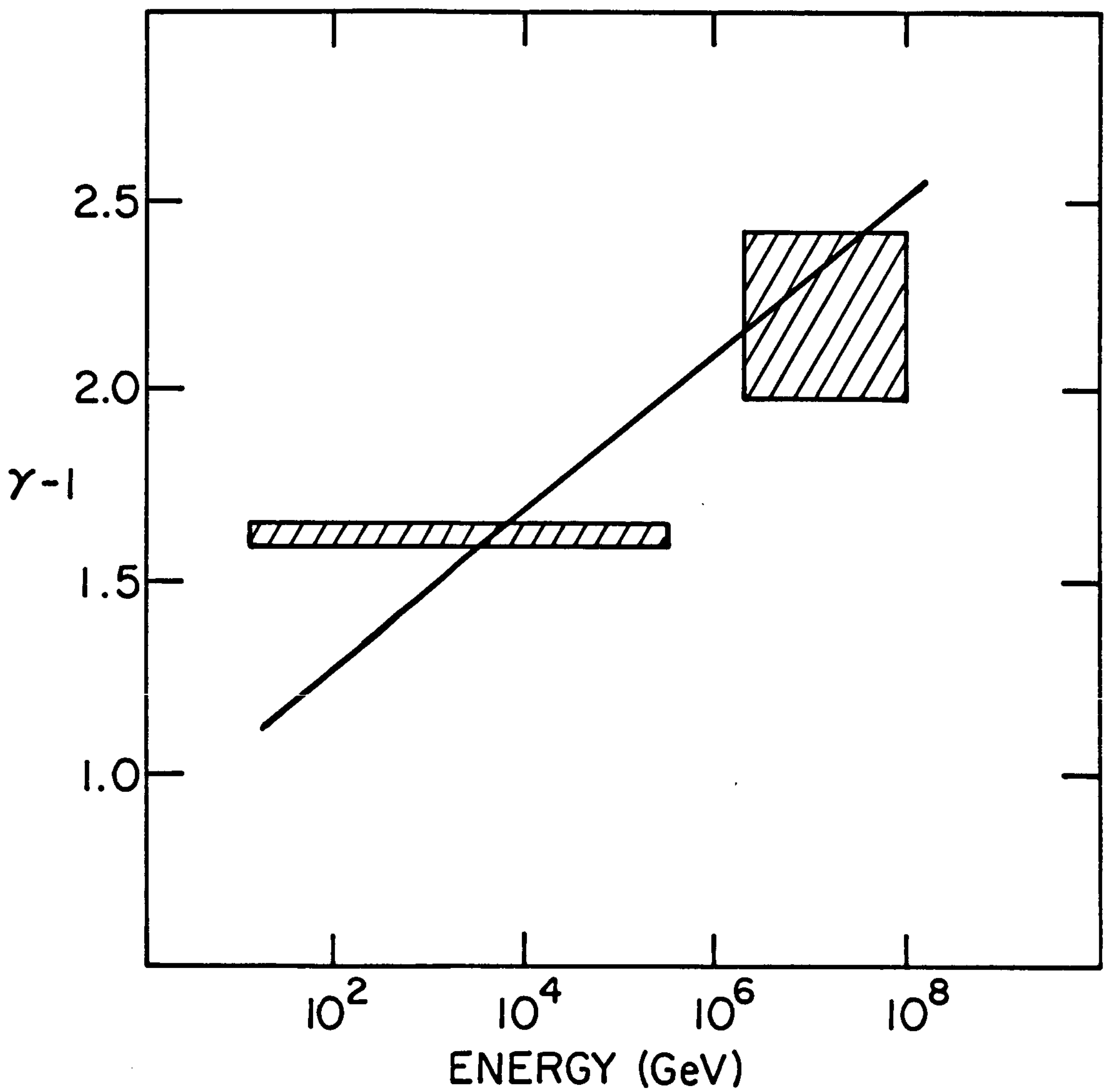


Fig. 3 THE POWER LAW EXPONENT OF THE INTEGRAL ENERGY SPECTRUM AS A FUNCTION OF ENERGY OF COSMIC RAY PROTONS.

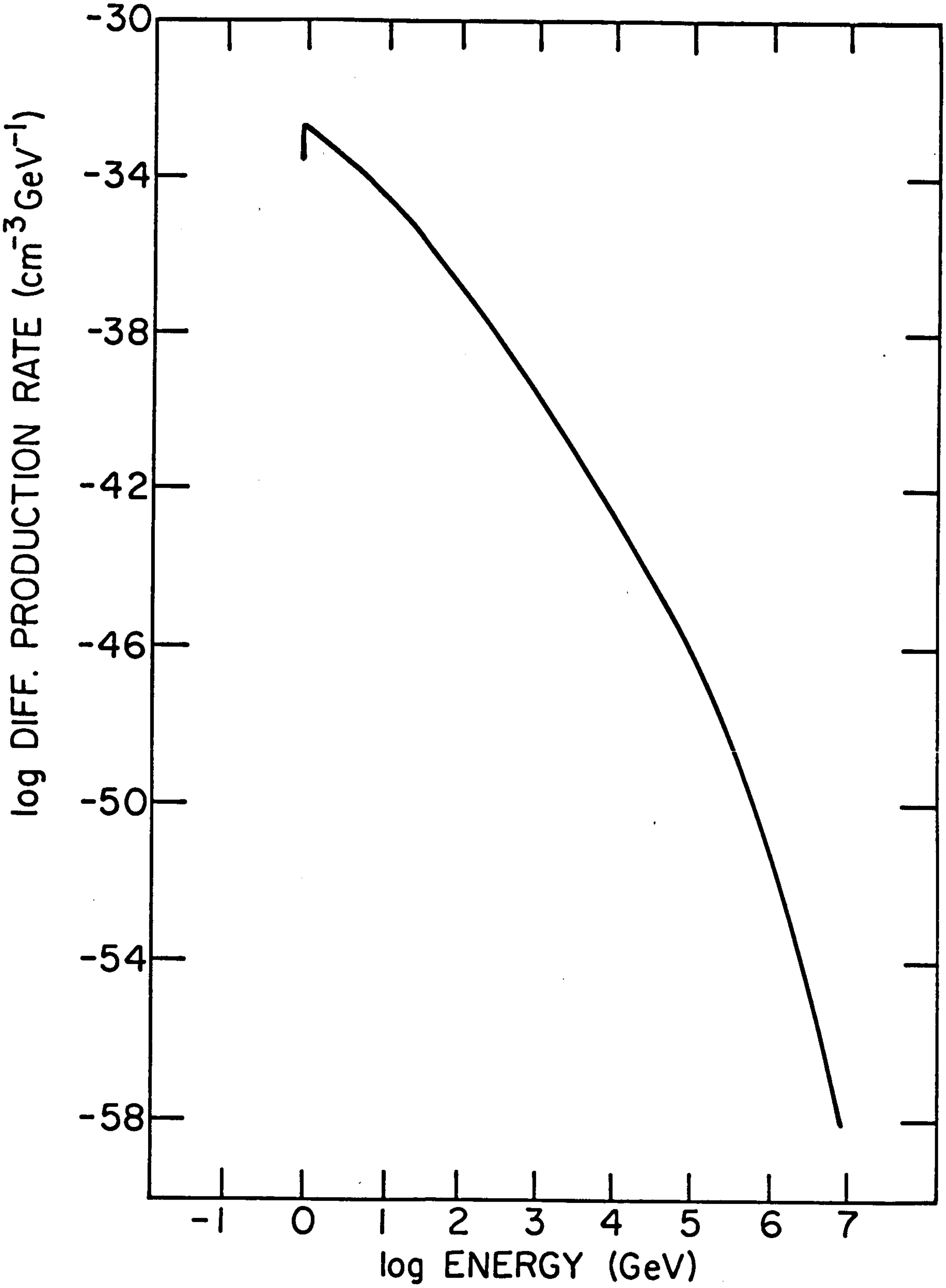


Fig. 4 THE ANTIPROTON INJECTION DIFFERENTIAL ENERGY SPECTRUM FOR $\bar{\rho} = 10^{-2}$ atoms/$cm^3$.

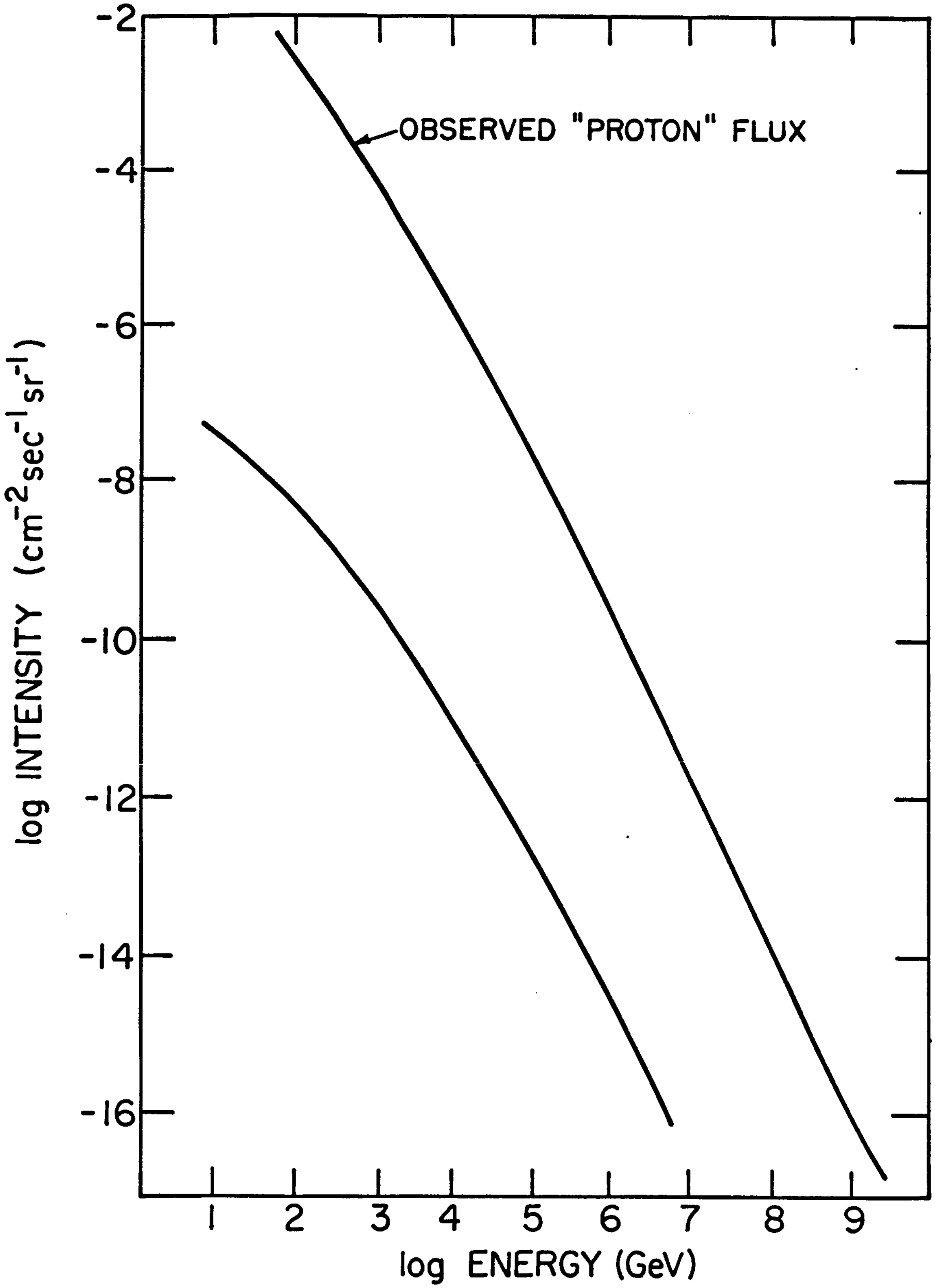


Fig. 5 THE ANTIPROTON INTEGRAL ENERGY SPECTRUM FLUX FOR CASE I.

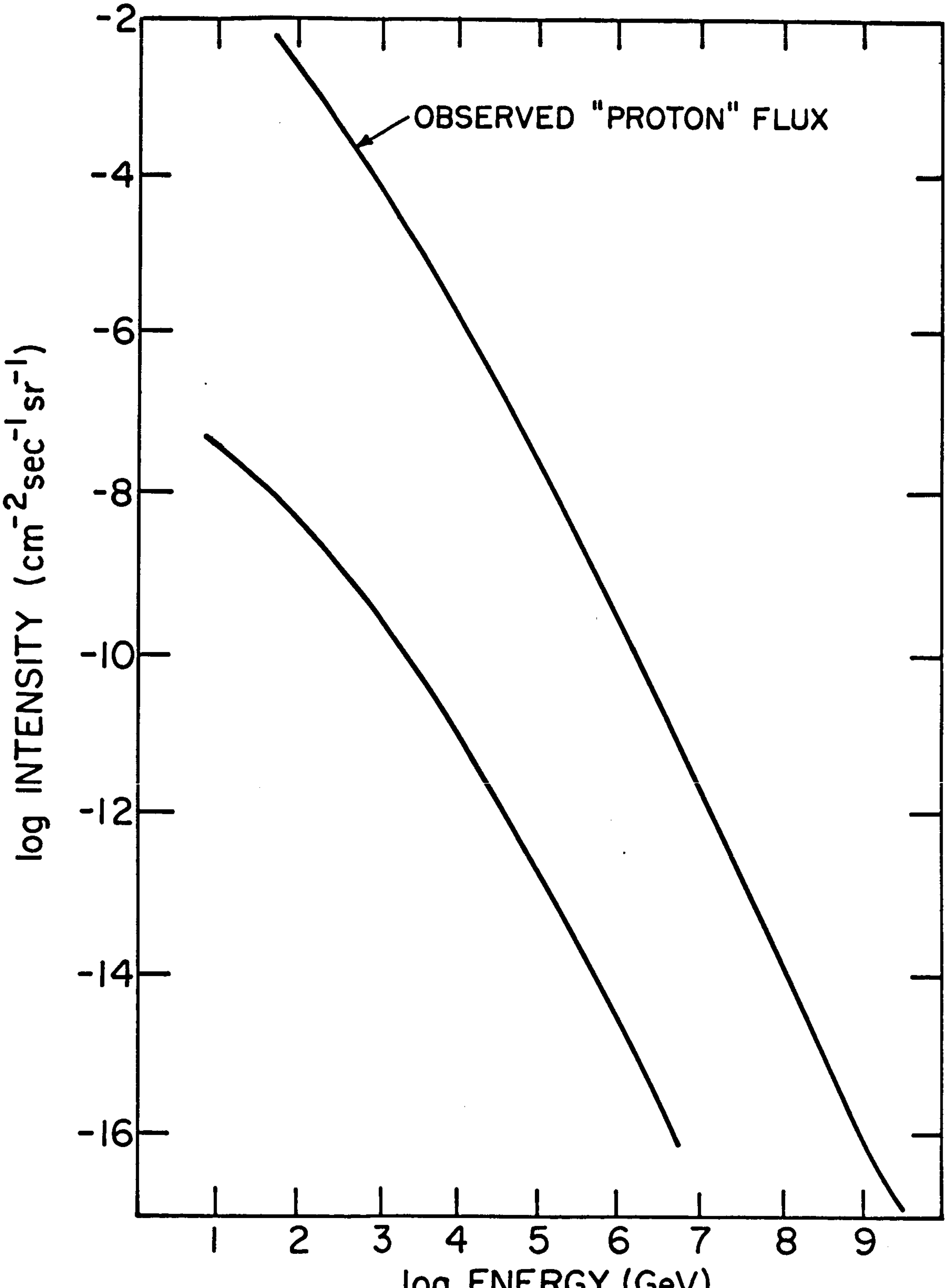


Fig. 6 THE ANTIPROTON INTEGRAL ENERGY SPECTRUM FLUX FOR CASE II.

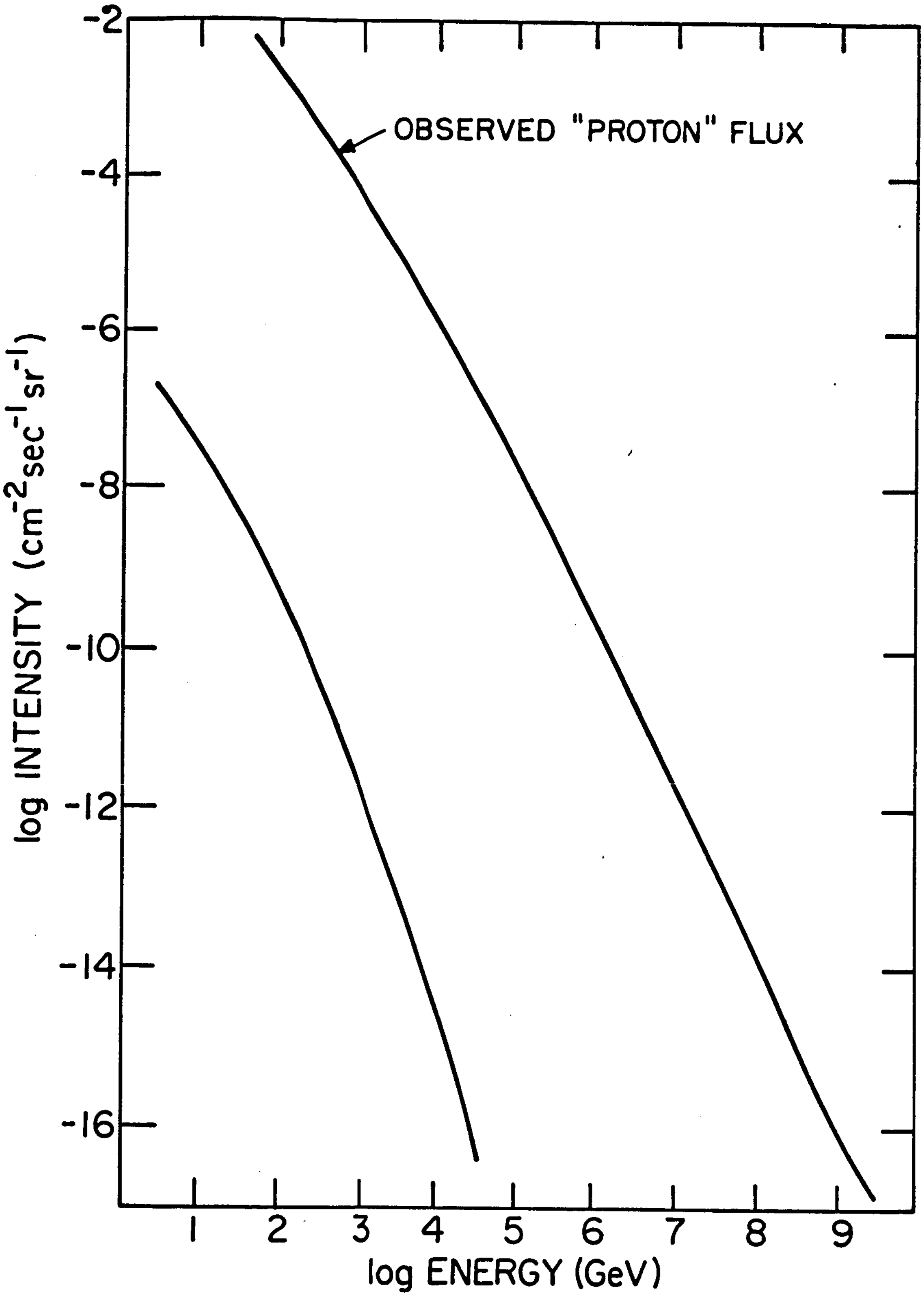


Fig. 7 THE ANTIPROTON INTEGRAL ENERGY SPECTRUM FLUX FOR CASE III.